

# Estimating the Spins of Stellar-Mass Black Holes

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We describe a program that we have embarked on to estimate the spins of stellar-mass black holes in X-ray binaries. We fit the continuum X-ray spectrum of the radiation from the accretion disk using the standard thin disk model, and extract the dimensionless spin parameter  $a_* = a/M$  of the black hole as a parameter of the fit. We have obtained results on three systems, 4U 1543-47 ( $a_* = 0.7 - 0.85$ ), GRO J1655-40 ( $0.65 - 0.8$ ), and GRS 1915+105 ( $0.98 - 1$ ), and have nearly completed analysis of two additional systems. We anticipate expanding the sample of spin estimates to about a dozen over the next several years.

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## 1. Introduction

The first black hole (BH), Cygnus X-1, was identified and its mass estimated in 1972. We now know of about 40 stellar-mass black holes in X-ray binaries in the Milky Way and neighboring galaxies. The masses of 21 of these, which range from  $\sim 5 - 15 M_\odot$ , have been measured by observing the dynamics of their binary companion stars (Remillard & McClintock 2006; Orosz et al. 2007). In addition, it has become clear that virtually every galaxy has a supermassive black hole with  $M \sim 10^6 - 10^{10} M_\odot$  in its nucleus. A few dozen of these supermassive BHs have reliable mass estimates, which have been obtained via dynamical observations of stars and gas in their vicinity (Begelman 2003).

With many mass measurements now in hand, the next logical step is to measure spin. This would mark a major milestone since, once we have both a BH's mass and spin, we will have achieved a complete description of the object. Furthermore, spin is arguably the more important parameter. Mass simply supplies a scale, whereas spin changes the geometry and fundamentally conditions the ways in which a BH interacts with its environment.

Unfortunately, spin is much harder to measure than mass. The effects of spin are revealed only in the regime of strong gravity close to the hole, where the sole probe available to us is the accreting gas. Thus, we must make accurate observations of the radiation emitted by the inner regions of the accretion disk, and we must have a reliable model of the emission. Until recently, there was no credible measurement of BH spin.

The situation has changed within the last couple of years. Following up on the pioneering work of Zhang, Cui & Chen (1997), the first breakthrough came with estimates of the spin parameter  $a_* \equiv a/M$  reported by our group (see Table 1) for three stellar-mass BHs (Shafee et al. 2006; McClintock et al. 2006): GRO J1655-40, 4U 1543-47, and GRS 1915+105. These spin estimates were obtained by modeling the continuum X-ray spectrum from the accretion disk surrounding the BHs. Following our work, the spin of a supermassive BH was estimated by an independent method, modeling the profile of the Fe K line (Brenneman & Reynolds 2006).

This paper is organized as follows. In §2 we describe the continuum-fitting method and comment on our efforts to establish our methodology. In §3 we review the extensive evidence for the existence of a stable inner accretion-disk radius, which provides a strong empirical foundation for the continuum-fitting method of determining spin. The impor-

TABLE 1  
Spin Estimates of Stellar-Mass Black Holes

BH Binary System	$M/M_{\odot}$	$a_*$	Reference
4U 1543–47	$9.4 \pm 1.0$	$0.7 - 0.85$	Shafee et al. (2006)
GRO J1655–40	$6.30 \pm 0.27$	$0.65 - 0.8$	Shafee et al. (2006)
GRS 1915+105	$14 \pm 4.4$	$0.98 - 1$	McClintock et al. (2006)

tance of measuring spin is briefly described in §4. In §5 we discuss work in progress and future prospects, and we offer our conclusions.

## 2. The Method: Fitting the X-ray Continuum Spectrum

A definite prediction of relativity theory is the existence of an innermost stable circular orbit (ISCO) for a test particle orbiting a BH. Once a particle is inside this radius, it suddenly plunges into the hole. Gas in a geometrically thin accretion disk has negligible pressure support in the radial direction and behaves for many purposes like a test particle. Thus, the gas spirals in (through the action of viscosity) via a series of nearly circular orbits until it reaches the ISCO, at which point it plunges into the BH. In other words, the disk is effectively truncated at an inner edge located at the ISCO.

In our method, we estimate the radius of the inner edge of the disk by fitting the X-ray continuum spectrum and identify this radius with  $R_{\text{ISCO}}$ , the radius of the ISCO. Since the dimensionless ratio  $\xi \equiv R_{\text{ISCO}}/(GM/c^2)$  is solely a monotonic function of the BH spin parameter  $a_*$  (Fig. 1), knowing its value allows one immediately to infer the BH spin parameter  $a_*$ . The variations in  $R_{\text{ISCO}}$  are large: e.g., for a BH of  $10M_{\odot}$ ,  $R_{\text{ISCO}}$  ranges from 90 km down to 15 km as  $a_*$  increases from 0 to unity, which implies that we should in principle be able to estimate  $a_*$  with good precision.

The idealized thin disk model (Novikov & Thorne 1973) describes an axisymmetric radiatively-efficient accretion flow in which, for a given BH mass  $M$ , mass accretion rate  $\dot{M}$  and BH spin parameter  $a_*$ , we can calculate precisely the total luminosity of the disk,  $L_{\text{disk}} = \eta \dot{M} c^2$ , where the radiative efficiency factor  $\eta$  is a function only of  $a_*$ , as well as the profile of the radiative flux  $F_{\text{disk}}(R)$  emitted as a function of radius  $R$ . Moreover, the accreting gas is optically thick, and the emission is thermal and blackbody-like, making it straightforward to compute the spectrum of the emission. Most importantly, as discussed above, the inner edge of the disk is located at the ISCO of the BH space-time. By analyzing the spectrum of the disk radiation and combining it with knowledge of the distance  $D$  to the source and the mass  $M$  of the BH, we can obtain  $a_*$ . This is the principle behind our method of estimating BH spin, which was first described by Zhang et al. (1997; see also Gierliński, Maciolek-Niedzwiecki & Ebisawa 2001).

In practice, as we describe below, the method involves fitting X-ray spectral data to a fully relativistic model of the disk emission and obtaining  $a_*$  as a fit parameter. However, one can understand the method qualitatively by noting that it effectively seeks to measure

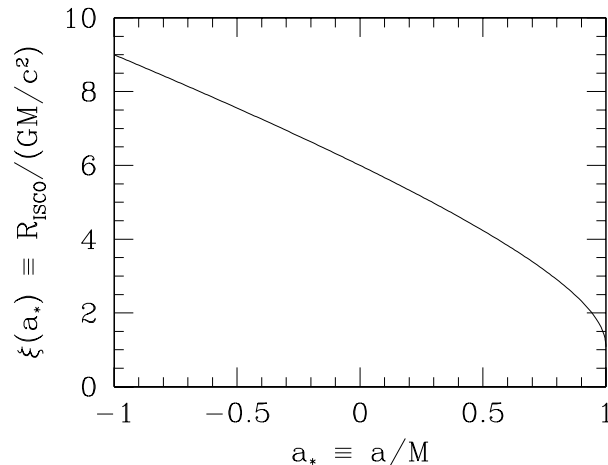


FIGURE 1. Shows the dependence of the quantity,  $\xi = R_{\text{ISCO}}/(GM/c^2)$ , on the BH spin parameter,  $a_* \equiv a/M = cJ/GM^2$ , where  $M$  and  $J$  are the mass and angular momentum of the BH (Shapiro & Teukolsky 1983). The spin parameter is restricted to the range  $-1 \leq a_* \leq 1$ ; negative values correspond to the BH counter-rotating with respect to the orbit.

the radius of the ISCO. Before discussing how this is done, we remind the reader how one measures the radius  $R_*$  of a star. Given the distance  $D$  to the star, the radiation flux  $F_{\text{obs}}$  received from the star, and the temperature  $T$  of the continuum radiation, the luminosity of the star is given by

$$L_* = 4\pi D^2 F_{\text{obs}} = 4\pi R_*^2 \sigma T^4. \quad (2.1)$$

Thus, from  $F_{\text{obs}}$  and  $T$ , we can obtain the solid angle  $\pi(R_*/D)^2$  subtended by the star, and if the distance is known, we immediately obtain the stellar radius  $R_*$ . Of course, for accurate results we must allow for limb darkening and other non-blackbody effects in the stellar emission by computing a stellar atmosphere model.

The same principle applies to an accretion disk, but with some differences. First, since  $F_{\text{disk}}(R)$  varies with radius, the radiation temperature  $T$  also varies with  $R$ . But the precise variation is known for the idealized thin disk, so it is easily incorporated into the model. Second, since the bulk of the emission is from the inner regions of the disk, the effective area of the radiating surface is directly proportional to the square of the disk inner radius,  $A_{\text{eff}} = CR_{\text{ISCO}}^2$ , where the constant  $C$  is known. Third, the observed flux  $F_{\text{obs}}$  depends not only on the luminosity and the distance, but also on the inclination  $i$  of the disk to the line-of-sight<sup>†</sup>. Allowing for these differences, one can write a relation for the disk problem similar in spirit to eq. (2.1), but with additional geometric factors that are readily calculated from the disk model. Therefore, in analogy with the stellar case, given  $F_{\text{obs}}$  and a characteristic  $T$  (from X-ray observations), one obtains the solid angle subtended by the ISCO:  $\pi \cos i (R_{\text{ISCO}}/D)^2$ . If we know  $i$  and  $D$ , we obtain  $R_{\text{ISCO}}$ , and if we also know  $M$ , we obtain  $a_*$ . This is the basic idea of the method.

We note in passing that for the method to succeed it is essential to have accurate measurements of the BH mass  $M$ , inclination of the accretion disk  $i$ , and distance  $D$  as

<sup>†</sup> We assume that the spin of the BH is approximately aligned with the orbital angular momentum vector of the binary; there is no strong contrary evidence despite the often-cited examples of GRO J1655-40 and SAX J1819.3-2525 (see §2.2 in Narayan & McClintock 2005).

inputs to the continuum-fitting process (Shafee et al. 2006; McClintock et al. 2006). This dynamical work is not discussed here, although roughly half of our total effort is directed toward securing these dynamical data (e.g., Orosz et al. 2007).

Given accurate information on  $M$ ,  $i$  and  $D$ , there are three main issues that must be dealt with before applying the method:

- (1) We must carefully trace rays from the surface of the orbiting disk to the observer in the Kerr metric of the rotating BH in order to compute accurately the observed flux and spectrum. To this end, our group has developed a model called KERRBB (Li et al. 2005) which has been incorporated into XSPEC (Arnaud 1996) and is now publicly available for fitting X-ray data.
- (2) We need an accurate model of the disk atmosphere for computing the spectral hardening factor  $f$  (see §4). We use the advanced models of our collaborator Shane Davis (Davis et al. 2005) and this element is thus well in hand. Specifically, we have computed tables of  $f$  versus  $L/L_{\text{Edd}}$  for a wide range of models. Further, we have incorporated these into a new version of KERRBB dubbed KERRBB2 (McClintock et al. 2006), which allows us to fit directly for the spin parameter  $a_*$  and the mass accretion rate  $\dot{M}$ .
- (3) Most importantly, the accretion disk around the BH must be well described by the standard geometrically-thin and optically-thick disk model, whose validity is assumed by KERRBB and KERRBB2. To ensure this, we restrict our attention strictly to observations in the thermal state (optically thick emission) and limit ourselves to luminosities below 30% of the Eddington limit (McClintock et al. 2006; Shafee, Narayan & McClintock 2007).

Beyond these three issues, we must ultimately push theory to its limits in order to understand accretion processes near the ISCO and to obtain the most accurate model of  $F_{\text{disk}}(R)$  that can be achieved (see §3).

For a full description of the mechanics of our current continuum-fitting methodology, we refer the reader to §4 in McClintock et al. (2006). In brief, we first select rigorously-defined thermal-state X-ray data (§4; Remillard & McClintock 2006). We then fit the broadband X-ray continuum spectrum using our fully relativistic model of a thin accretion disk (KERRBB2) in Kerr space-time, which includes all relativistic effects (Li et al. 2005) and an advanced treatment of spectral hardening (§4; Davis et al. 2005). The model also includes self-irradiation of the disk (“returning radiation”), the effects of limb darkening, and the effect of a torque of any magnitude at the inner edge of the disk, although our published results are based on zero torque, which is justified in Shafee et al. (2007). As noted above, our new hybrid code KERRBB2 allows us to fit directly for the two parameters of interest: the spin  $a_*$  and the mass accretion rate  $\dot{M}$ . Using the known radiative efficiency factor  $\eta$  of the disk for a given  $a_*$ , and the fitted value of  $\dot{M}$ , we compute for each observation the Eddington-scaled luminosity,  $L/L_{\text{Edd}}$ , and consider only those observations for which  $L/L_{\text{Edd}} < 0.3$  (§3; Shafee et al. 2007). Finally, we present our results in the form of plots of  $a_*$  versus  $\log(L/L_{\text{Edd}})$ .

As an example, Figure 2 shows our results on GRS 1915+105 (McClintock et al. 2006). Over the luminosity range  $L/L_{\text{Edd}} < 0.3$ , the data are consistent with a single value of  $a_*$  close to unity. Allowing for statistical errors and uncertainties in the input values of  $M$ ,  $i$  and  $D$ , we estimate  $a_*$  to lie in the range  $0.98 - 1$  (Table 1). For luminosities closer to Eddington, the  $a_*$  estimates obtained using our method are lower, as also found by Middleton et al. (2006), who analyzed three observations with luminosities between  $0.4L_{\text{Edd}}$  and  $1.4L_{\text{Edd}}$  (cf. McClintock et al. 2006, Fig. 12). Neither the cause for the decrease nor its magnitude are presently understood. However, it is not surprising that our model, which assumes a geometrically thin accretion disk, should fail at luminosities close to Eddington when the disk is likely to be very thick.

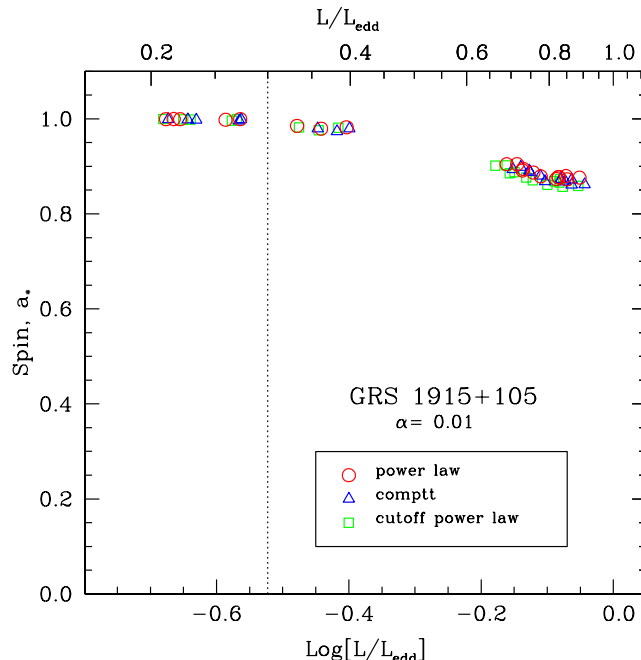


FIGURE 2. Shows the estimated spin parameter  $a_*$  of the BH in GRS 1915+105, as a function of the Eddington-scaled luminosity  $L/L_{\text{Edd}}$ . The spectral data were analyzed using KERRBB2 combined with three different models of the high energy Comptonized radiation (shown by different symbols). For  $L/L_{\text{Edd}} < 0.3$  (to the left of the vertical dotted line), all the estimates of  $a_*$  are consistent with a value nearly equal to unity. The result is insensitive to the precise Comptonization model used in the analysis. (Taken from McClintock et al. 2006).

The results we published on 4U1543–47 and GRO J1655–40 in Shafee et al. (2006) were obtained with KERRBB. We have re-analyzed the same data using KERRBB2, which gives a slightly larger range of uncertainty for the derived values of  $a_*$ . The spin values listed in Table 1 correspond to the more recent analysis.

### 3. Establishing the Continuum-Fitting Method

Given our straightforward methodology and our in-depth experience in determining the spins of three BHs, we are confident that we can achieve our goal of amassing a total of a dozen or so measurements of BH spin during the next 3–4 years. Equally important, however, are our efforts to demonstrate that our methodology is sound. The largest systematic error in the BH spin estimates reported so far arise from uncertainties in the validity of the disk model we employ. Thus, it is obviously crucial to pursue detailed theoretical studies of the physics of BH accretion flows near the ISCO.

Recently, we obtained encouraging preliminary results (Shafee et al. 2007) based on a hydrodynamic study showing that the errors in our spin estimates due to viscous torque and dissipation near the ISCO are quite modest for disk luminosities  $\lesssim 30\%$  of the Eddington limit. This is the luminosity limit that we had already adopted in our earlier work (McClintock et al. 2006). We are presently working to extend these hydro models to full GR MHD, where magnetic stresses may possibly cause important deviations from the standard thin disk model (e.g., Krolik 1999; Gammie 1999; Krolik & Hawley 2002).

In addition to this fundamental theoretical work, we are engaged in a broader effort

to assess all scenarios that can ultimately impact upon our estimates of BH spin. Two examples: (1) With J. C. Lee, we are examining the possible effects of warm absorbers (i.e., photoionized gas) on our spin estimates via an analysis of HETG grating spectra; and (2) we are in the process of making a stringent test of our spin model by obtaining a VLBA parallax distance and improved radial velocities for the microquasar GRS 1915+105 (see §6.4 in McClintock et al. 2006).

#### 4. A Basis for Optimism

Among the several spectral states of accreting BHs, the *thermal state* (see Table 2 in Remillard & McClintock 2006), formerly known as the high soft state, is central to the work proposed here. A feature of this state is that the X-ray spectrum is dominated by a soft blackbody-like component which is emitted by (relatively) cool optically-thick gas in the accretion disk. In addition, there is a minor nonthermal tail component of emission, which probably originates from a hot optically-thin corona. In practice, this poorly-understood Comptonized component of emission contributes  $\lesssim 10\%$  of the flux in a 2–20 keV band (e.g., *RXTE*) and an even much smaller fraction in an 0.5–10 keV band (e.g., *ASCA* and *Chandra*), which captures nearly all of the  $\sim 1$  keV thermal spectrum. Thus, the only spectra we consider – thermal-state spectra – are largely free of the uncertain effects of Comptonization (e.g., Fig. 2). These observed spectra are believed to match very closely the classic thin accretion disk models of the early 1970s (Shakura & Sunyaev 1973; Novikov & Thorne 1973).

There is a long history of evidence suggesting that fitting the X-ray continuum is a promising approach to measuring BH spin. This history begins in the mid-1980s with the simple non-relativistic multicolor disk model (Mitsuda et al. 1984; Makishima et al. 1986), which returns the color temperature  $T_{\text{in}}$  at the inner-disk radius  $R_{\text{in}}$ . In their review paper on BH binaries, Tanaka & Lewin (1995) summarize examples of the steady decay (by factors of 10–100) of the thermal flux of transient sources during which  $R_{\text{in}}$  remains quite constant (see their Fig. 3.14). They remark that the constancy of  $R_{\text{in}}$  suggests that this fit parameter is related to the radius of the ISCO. More recently, this evidence for a constant inner radius in the thermal state has been presented for a number of sources via plots showing that the bolometric luminosity of the thermal component is approximately proportional to  $T_{\text{in}}^4$  (Kubota, Makishima, & Ebisawa 2001; Kubota & Makishima 2004; Gierliński & Done 2004; Abe et al. 2005; McClintock et al. 2007).

We now demonstrate that the case for the constancy of the inner disk radius is further strengthened if one considers the effects of spectral hardening, which we determine via the state-of-the-art disk atmosphere models of Davis et al. (2005). At the high disk temperatures typically found in BH disks ( $T_{\text{in}} \sim 10^7$  K), non-blackbody effects are important and one replaces  $T_{\text{in}}$  by the effective temperature  $T_{\text{eff}} = T_{\text{in}}/f$ , where  $f$  is a “spectral hardening factor” (Shimura & Takahara 1995; Merloni, Fabian, & Ross 2000; Davis et al. 2005). In Figure 3, we illustrate the effects of spectral hardening on the relationship between luminosity and temperature for two BH transients (see also Davis, Done, & Blaes 2006). The figure extends results that are presented in Figure 8 in McClintock et al. (2007). The top two panels show the Eddington-scaled luminosities of the two BH transients during their entire outburst cycles. The bold plotting symbols denote the rigorously-defined thermal-state data (see Table 2 in Remillard & McClintock 2006). In the lower panels, we consider only these thermal-state spectral data, and we ignore the remaining data that are strongly Comptonized and for which the models are very uncertain.

Panels *b* show plots of Eddington-scaled luminosity versus the color temperature  $T_{\text{in}}$ ;

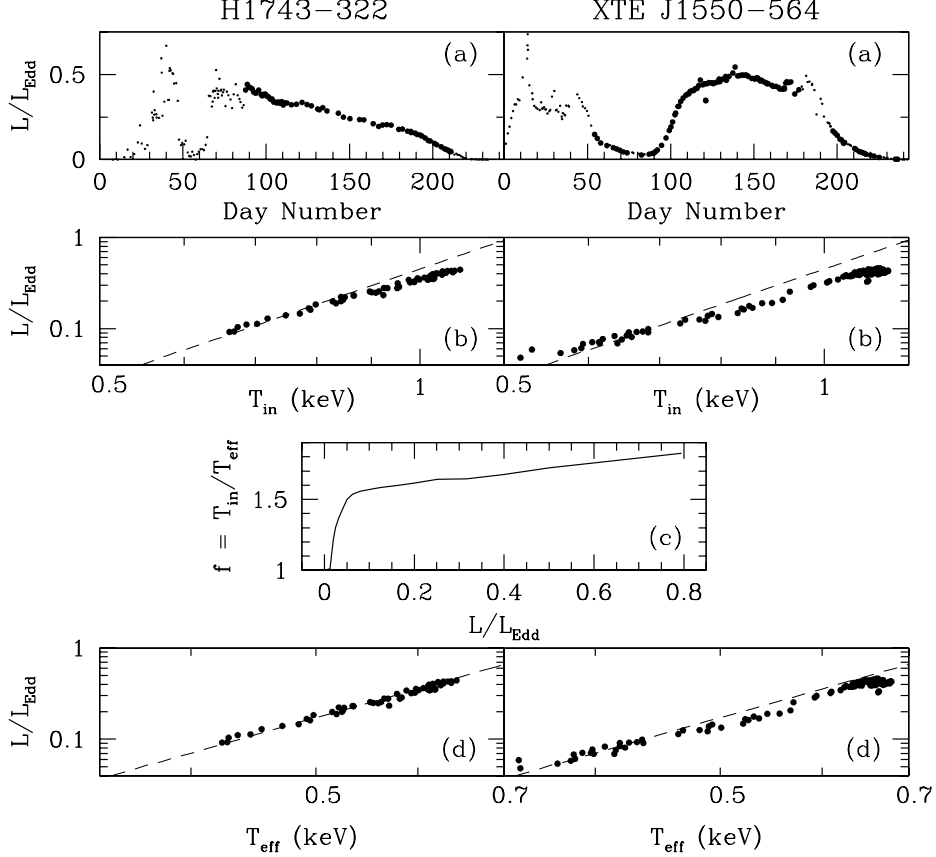


FIGURE 3. Evidence for the constancy of the inner disk radius and an illustration of the effects of spectral hardening. Shown are thermal-state data collected for H1743-322 in 2003 and XTE J1550-564 in 1998–1999 in hundreds of pointed observations using the *RXTE* PCA detector (McClintock et al. 2007). (a) The evolution of the luminosities of the two transients throughout their complete 8-month outburst cycles. The luminosities are scaled to the Eddington limit; for mass and distance estimates, see McClintock et al. (2007). (b) Luminosity versus the color temperature; the log-log slope of the dashed line is 4. (c) The spectral hardening factor  $f \equiv T_{\text{in}}/T_{\text{eff}}$  versus luminosity computed from the disk atmosphere model of Davis et al. (2005) using BHSPEC in XSPEC (Arnaud 1996). This model was computed for a PCA response matrix in the 2–20 keV band,  $M = 10M_{\odot}$  and  $i = 70^{\circ}$  (McClintock et al. 2007), and  $a_* = 0.5$ . The model depends only weakly on the assumed value of the spin parameter. (d) Luminosity versus the effective temperature  $T_{\text{eff}} = T_{\text{in}}/f$ , derived from the model results shown in panel c. Note how the data here hug the dashed  $T^4$  line much more closely than in panels b.

the dashed lines show an  $L/L_{\text{Edd}} \propto T_{\text{in}}^4$  relation (McClintock et al. 2006). Note that the observed luminosity rises more slowly than  $T_{\text{in}}^4$ , which appears to suggest that  $R_{\text{in}}$  is not constant. Panel c shows an appropriate model of the spectral hardening factor  $f$  as a function of luminosity. Using this relationship, we replotted the luminosity data shown in panels b versus  $T_{\text{eff}}$ , thereby obtaining the results shown in panels d. Here one

finds that the luminosity is closely proportional to  $T_{\text{eff}}^4$ , which provides strong evidence for the presence of a *stable inner disk radius*. Obviously, this non-relativistic analysis cannot provide a secure value for the radius of the ISCO nor even establish that this stable radius is the ISCO. Nevertheless, the presence of a fixed radius indicates that the continuum-fitting method is a well-founded approach to measuring BH spin.

## 5. Importance of Measuring Spin

In order to model the ways that an accreting BH can interact with its environment, one must know its spin. For example, the many proposals relating relativistic jets to BH spin (Blandford & Znajek 1977; Meier 2003; McKinney & Gammie 2004; Hawley & Krolik 2006) will remain mere speculation until sufficient data on BH spins have been amassed and models are tested and confirmed. Likewise, measurements of spin are comparably important for testing stellar-collapse models of Gamma-Ray Burst sources (Woosley 1993; MacFadyen & Woosley 1999; Woosley & Heger 2006). Knowledge of spin is also crucial for the development of gravitational-wave astronomy, and our Shafee et al. (2006) paper has already motivated the first computation of waveforms for coalescing BHs that includes the effects of spin (Campanelli, Lousto & Zlochower 2006). There are several other obvious applications of spin data, such as crucial input to models of BH formation and BH binary evolution (Lee, Brown & Wijers 2002; Brown et al. 2007) and to models of the powerful low-frequency QPOs (1–30 Hz) and complex, non-thermal BH states and their evolution (Remillard & McClintock 2006). Finally, we note that the high spins we have measured to date were very likely imparted to these BHs during the process of their formation (see §6.2 in McClintock et al. 2006).

## 6. Conclusions and Future Prospects

We have recently completed a thorough and precise dynamical study of the only known eclipsing BH, M33 X-7 (Pietsch et al. 2006), which is the most massive stellar BH known,  $M = 15.65 \pm 1.45 M_{\odot}$  (Orosz et al. 2007). Furthermore, the mass of the secondary star is  $M_2 = 70.0 \pm 6.9 M_{\odot}$ , which puts it among the most massive stars whose masses are well-determined. We are presently preparing a paper on the spin of this BH based on  $\sim 2$  Msec of *Chandra* ACIS data (Liu, McClintock, Narayan, et al.). We are also in the process of estimating the spin of XTE J1550–564 using *RXTE* PCA data, and we anticipate estimating the spins of more than half a dozen other stellar-mass BHs during the next 3–4 years.

An especially exciting prospect is the possibility of obtaining independent estimates of spin via either the Fe K line profile (Reynolds & Nowak 2003; Brenneman & Reynolds 2006; Miller et al. 2007) or high-frequency (50 – 450 Hz) QPOs (Török et al. 2005; Remillard & McClintock 2006), which are observed for some of these sources. Because spin is such a critical parameter, we and many others are planning to pursue vigorously these additional avenues, as this will provide arguably the best possible check on our results. Future X-ray polarimetry missions may provide yet an additional channel for measuring spin (e.g., Connors, Stark & Piran 1980).

We conclude with a list of questions that motivate us. What range of spins will we find? Will GRS 1915+105 stand alone, or will we find other examples of extreme spin? As we continue to refine our models and our measurements of  $M$ ,  $i$  and  $D$ , will we consistently find values of  $a_* < 1$ , or will we be challenged by apparent and unphysical values of the spin parameter that exceed unity? Will there be large differences in spin between the class of young, persistent systems with their massive secondaries (M33 X-7, LMC



X-1 and LMC X-3) and the ancient transient systems with their low-mass secondaries? What constraints will these spin results place on BH formation, evolutionary models of BH binaries, models of relativistic jets and gamma-ray bursts, etc.? What will be the implications of these spin measurements for the emerging field of gravitational-wave astronomy in the Advanced LIGO era? How will this new knowledge help shape the observing programs of *GLAST*, *Black Hole Finder Probe*, *Constellation-X*, and *XEUS*?

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